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Field Balancing Large Rotating Machinery

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PREFACE

One of the primary responsibilities of the Facilities Engineering Branch is to provide technical assistance to field maintenance personnel. Bulletins such as this are written to provide assistance and knowledge for field personnel to better perform plant operations. Also, it helps to minimize the number of field trips by Denver Office personnel.

Since the previous edition (No. 13A, 1946) of this bulletin is outdated and more technical than necessary, effort has been made to simplify the procedures and to update the bulletin.

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FIELD BALANCING
LARGE ROTATING MACHINERY

1. Purpose

This bulletin is intended as a guide to aid plant maintenance personnel in balancing large rotating machinery. Vibration literature[1] can be obtained for study in more technical detail. The objective of balancing is to reduce shaft vibration to a practical minimum. Reducing shaft vibrations generally reduces bearing loads and increases the service life.

1.1 - Turbine runners and pump impellers are usually dynamically balanced before they are installed and seldom require rebalancing. Normally, the rotating parts of generators and motors are balanced after installation. Maintenance work and slight shape changes with age - in some instances - can alter the balance of a generator or motor rotating parts enough to require rebalancing. This bulletin is concerned with correcting the unbalanced masses in rotors and drive shafts.

2. Vibration Sources

Large hydraulic units are subjected to many kinds of vibrations.

Vibration and unbalance can be caused by *mechanical, electrical, or hydraulic* problems. The unbalance of rotating parts is the most common cause of excessive unit vibration. Before attempting to balance the rotating parts of a unit, determine the possible sources of vibration and unbalance.

Mechanical vibration sources are:

- Bolted connections in the rotating parts should be checked for tightness
- Unit alinement of the rotor or runner
- Poor bearing conditions or position
- Foundation rigidity (flexure)

Check for possible electrical sources of vibration such as:

- Nonuniform air gap in the motor, generator, or exciter
- Short-circuited winding turns in the rotor poles
- Ellipticity of the rotor

The hydraulic passages of the unit should be checked for:

- A nonuniform pressure distribution over the surfaces of the turbine runner or pump impeller can cause hydraulic unbalance
- Obstructions in the spiral case or volute
- Debris between the vanes
- Incorrect vertical position of the runner or impeller (relative to distributor)

Excessive cavitation in the unit causes hydraulic unbalance.

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[1] R. P. Kroon, *Balancing of Rotating Apparatus II,* Journal of Applied Mechanics, vol. 11, No. 1, March 1944.

E. L. Thearle, *Dynamic Balancing of Rotating Machinery in the Field*, translation of the ASME, October 1934.

3. Theory of Balancing

Unbalanced masses in the rotating parts create a centrifugal force that causes the unit to vibrate. Field balancing consists of determining the *amplitude* (size) and *location* around the shaft (phase angle) of the unbalance and placing weights on the rotor to counter the unbalance.

3.1- As the rotor rotates (fig. 1), the unbalanced mass *m* tends to pull the rotor toward the bearings on the side the unbalance is located creating an apparent *high spot* on the shaft. At very low speeds, this *high spot* will be in phase with the unbalanced (heavy) mass. As speed increases, the *high spot* begins to lag the *heavy spot*.

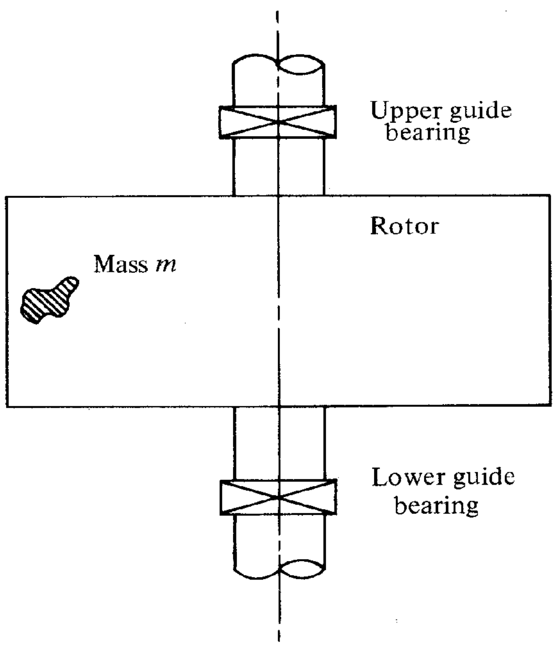


Figure 1.–Elevation of rotor having unbalanced mass.

3.2 - For hydroelectric units, the operating speed usually is less than the critical speed and the lag angle will be less than 90°.

3.3 - Figure 2 shows the relation between the unbalanced mass and the *high spot* at normal operating speed.

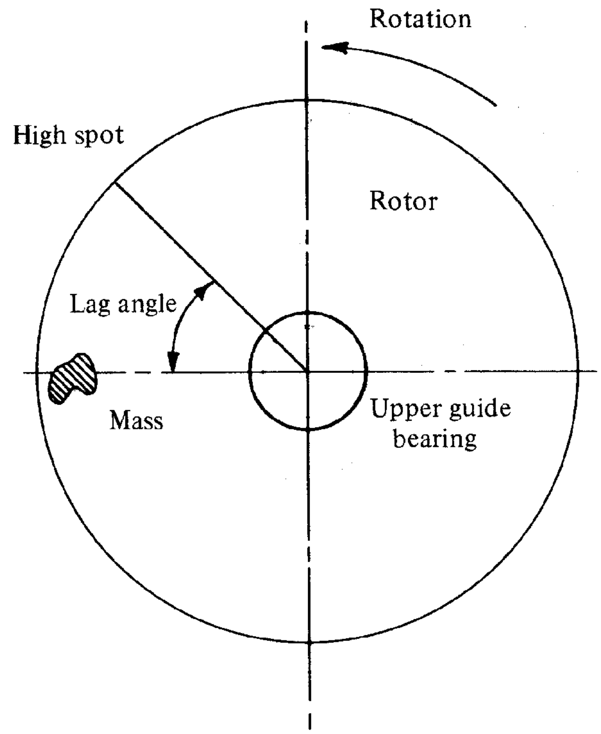


Figure 2.–Plan of rotor.

3.4 - When balancing, the machine is assumed to be linear; i.e., vibration amplitudes are in proportion to the forces causing them. For the purpose of this bulletin, the term *vibration amplitude* is synonymous with *shaft deflection* which is a measure of the unbalance present. The point that the vibration amplitude is maximum is called the high spot. Consequently, to balance the unit, it is necessary to determine the *vibration amplitude* and *location* of the shaft deflection caused by the unbalanced condition. With this information known, a balance weight is determined and positioned which gives a counter effect and balances the unit.

4. Instrumentation and Measurement

Several methods are used to locate and measure the unbalance. The most common method requires dial indicators and a stroboscope. Dial indicators are mounted, with the indicator stems in contact with the shaft, at locations where deflection measurements are desired. The shaft is marked and numbered with equally spaced segments. By using the stroboscope speed control, the scope is adjusted to flash at a number on the shaft and the dial indicator reading corresponding to that number is recorded. Dial indicator readings are recorded at each numbered location. The largest reading and its location are used in balancing techniques discussed later in this bulletin.

4.1 - A more recent method requires the use of a direct writing oscillograph and proximity indicators. The proximity indicators are calibrated to show the shaft deflection magnitude. The shaft is marked by cementing a steel shim at a known angular location - usually corresponding to No. 1 rotor arm. As the shim passes a proximity device, a characteristic mark will appear on the oscillograph record. This mark then can be used to locate the angular position of the maximum shaft deflection. This method has the advantage that - once set up - the deflection at every point around the shaft can be quickly and accurately measured and a written record is generated automatically.

4.2 - A seismic velocity transducer and vibration analyzer also can be used like the proximity indicator and oscillograph. The analyzer indicates visually the vibration amplitude and phase angle. Some analyzers often include an automatic chart recorder to provide a written record of the balancing.

4.3 - Regardless of the method used to obtain data, the measurements are the same. Shaft runout readings are taken simultaneously at (fig. 3):

- upper guide bearing
- lower guide bearing
- turbine guide bearing

The measurement points should be in the same vertical plane. Data from the upper and lower guide bearings are used to balance the unit. Data from the turbine guide bearing are monitored to assure that the rotor balancing is not adversely affecting the turbine guide bearing.

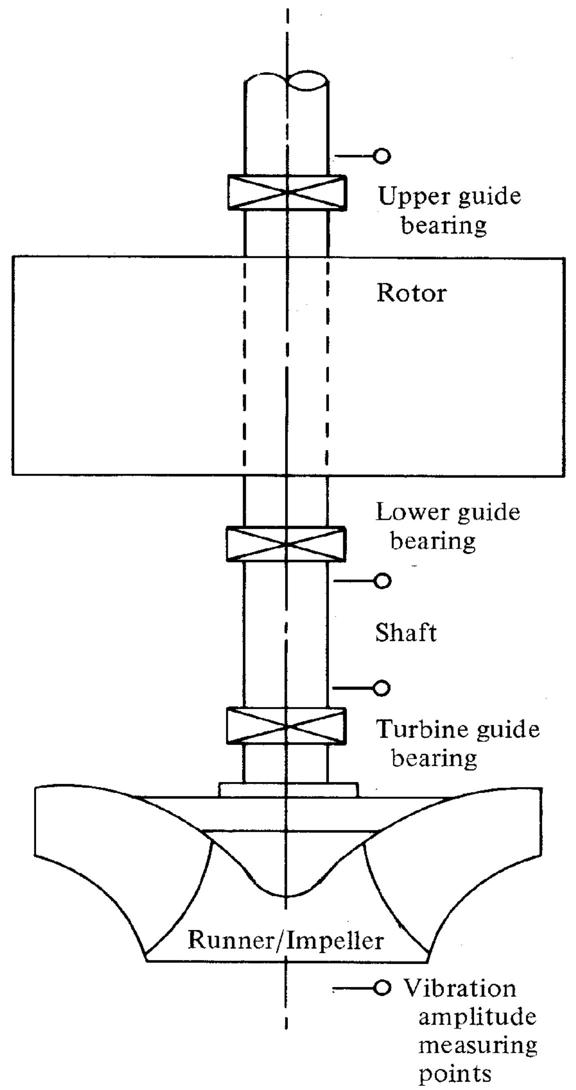


Figure 3.–Location of vibration amplitude measuring points.

4.4 - Three trial runs are made to obtain data necessary to balance a hydraulic unit. The first run is made in the as-found condition to determine the vibration amplitude and location of the existing shaft deflection.

For the second run, a trial weight is attached at the top of a rotor arm. The third run is made with the trial weight removed from the top of the rotor arm and attached to the bottom of the same rotor arm.

4.5 - Experience indicates that the trial weight should be approximately equal to the weight of the rotating parts divided by 10,000. Ideally, the weight should be located 180º plus the lag angle - in the direction of rotation - from the *high spot* to counter the unbalance. Therefore, the mass should be attached to the rotor arm nearest this location. In most cases, the lag angle will be unknown and in that event assume it to be 45º. Throughout this bulletin, angles are measured in the direction of unit rotation with respect to rotor arm No. 1 - the reference point.

5. Vector Techniques

The process of determining the required balance weights and locations can be simplified by using vectors to represent the unbalance. Definitions and examples are shown below to explain vector procedure.

5.1 - A vector is defined as a quantity that has both magnitude and direction. The direction of vectors is measured with respect to the same reference line.

5.2 - ***Example 1***: Measurements at the upper generator guide bearing indicate a shaft deflection of 0.006 inch at 30º from No. 1 rotor arm. The 0.006 inch is the magnitude of vibration amplitude and the 30º counterclockwise is the location (direction). Using a scale (such as one-fourth inch equals 0.001 inch), draw the deflection as a vector. The 0.006 inch at 30º has been scaled as vector ***OA***. Point ***O*** is the tail of the vector and point ***A*** is the head. Note that the arrowhead always points in the direction that the vector acts, in this case away from point ***O*** and towards A at an angle of 30º from the reference line. Thus, this vector can be represented by the notation ***OA*** and is not the same as ***AO***.

6. Vector Addition

A vector can be added to another vector (original) by using graphical techniques. Draw the vector to be added using the head of the original vector as the starting point. A vector drawn from the tail of the original vector to the head of the added vector is the graphical sum (resultant) of the two vectors.

6.1 - ***Example 2***: A vector ***OB*** having a magnitude of 0.005 inch and a direction of 150º is added to vector ***OA*** in figure 4.

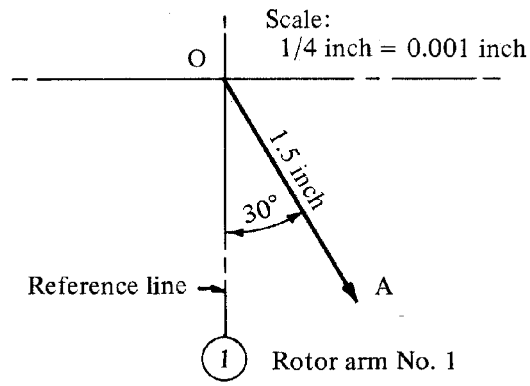


Figure 4.–Vector *OA* direction is 30° having a magnitude of 0.006 inch.

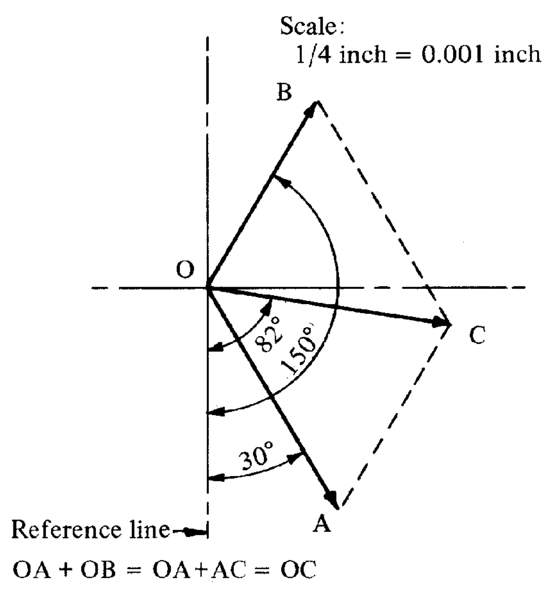


Figure 5.–Vector addition.

Both vectors are shown in figure 5. To add two vectors, translate vector ***OB*** as described above and shown in figure 5 as ***AC***. Note ***AC*** equals ***OB***. Complete the triangle by drawing a vector ***OC*** from the tail of ***OA*** to the head of ***AC***. This vector ***OC*** is the graphical sum of ***OA*** plus ***OB***. The magnitude and direction of ***OC*** can be scaled as 0.0056 inch and 82°, respectively.

Note that the term "graphical sum" has been stressed. Vectors cannot be added in the usual sense without complicating this discussion. Thus, 0.005 plus 0.006 equals 0.011 in ordinary addition; however, graphical vector addition yields:

$$0.005 \underline{/150^\circ} + 0.006 \underline{/30^\circ} = 0.0056 \underline{/82^\circ}$$

The notation 0.005 $\underline{/150^\circ}$ describes a vector having magnitude 0.005 inch and in a direction 150° from some reference line or axis.

7. Vector Subtraction

To subtract vector ***B*** from vector ***A***, add the negative of vector ***B*** to vector ***A***. The negative of a vector is equal in magnitude to the vector but opposite in direction.

7.1 - ***Example 3***: In figure 6, vector ***OB*** is to be subtracted from vector ***OA.*** The vector **- *OB*** is the negative of ***OB***. Placing the tail of **- *OB*** at the head of ***OA*** and completing the triangle yields the resultant of ***OA*** **-** ***OB*** which is ***OD***. Note that ***OA*** **-** ***OB*** is equal in magnitude but opposite in direction to ***OB*** **-** ***OA***. The magnitude ***OD*** is:

$$0.006 \underline{/30^\circ} - 0.005 \underline{/150^\circ} =$$
$$0.006 \underline{/30^\circ} + 0.005 \underline{/330^\circ} = 0.0095 \underline{/3^\circ}$$

8. Component Vectors

Often it is necessary to reduce a given vector into two component vectors of known directions. These vectors will produce the same effect and, when added, yield the given vector. To graphically obtain the magnitudes of the two component vectors, construct a parallelogram where two sides are parallel to the known directions and also which has the given vector as the diagonal. The two sides are the component vectors.

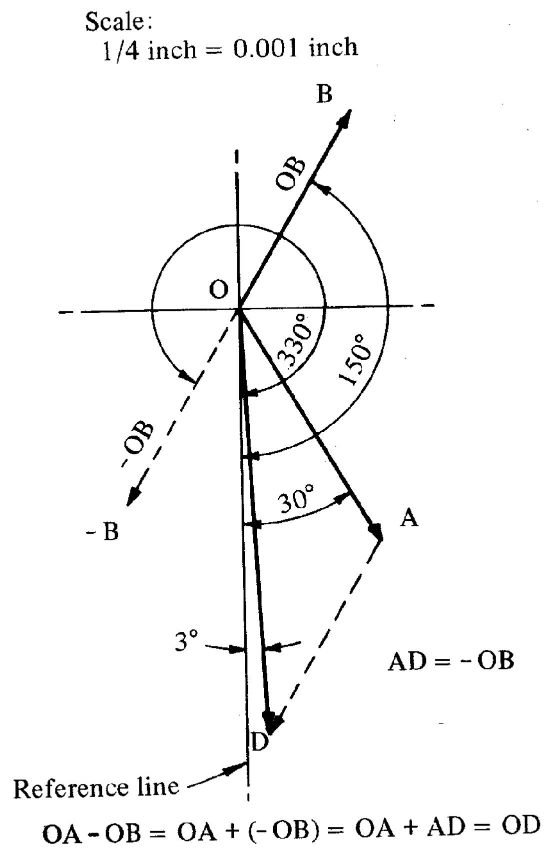


Figure 6.—Vector subtraction.

8.1 - ***Example 4***: It is determined that a 50-pound weight placed 15° from *rotor arm* No. 1 (at 0°) will balance a rotor. However, the closest locations for installing weights are at 0° and 60° (rotor arm No. 2). Therefore, two weights must be installed at 0° and 60° that will have the resultant effect of a 50-pound weight at 15°.

8.2 - The 50-pound weight at 15° is scaled as the vector ***OE*** (fig. 7). To reduce ***OE*** into a pair of vectors located at 0° and 60° (fig. 7) construct a parallelogram with sides at 0° and 60° and ***OE*** as the diagonal at 15°. Beginning at the head of given vector ***OE***, draw a line parallel to the 0° line

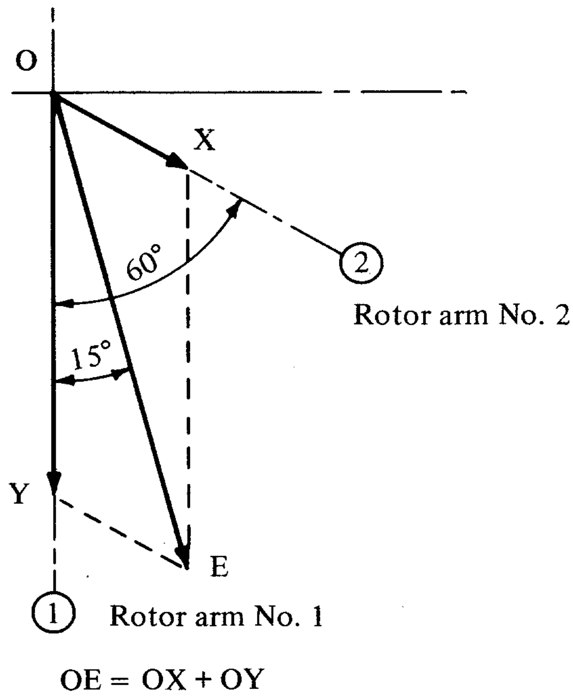


Figure 7.–Component vectors.

(rotor arm No. 1) and extend it to intersect the 60° line (rotor arm No. 2). This intersection is ***X***. Also, beginning at the head of ***OE***, draw a line parallel to the 60° line and extend it to intersect the 0° line. This intersection is ***Y***. The pair of vectors ***OX*** and ***OY*** are components to the given vector ***OE***. Therefore, the rotor should balance by placing a weight equal in magnitude to ***OX*** at 60° and a weight equal in magnitude to ***OY*** at 0°. By scaling, ***OX*** and ***OY*** are 15 and 42 pounds, respectively.

9. Static Balancing

Static or *single plane* balancing-can be used to balance large motors and generators. In such cases, the balance weight has the same effect when placed on either the top or bottom of the rotor arm. If the results of the trial runs show only small differences in vibration amplitude and phase angle (lag angle), when the weight is changed from the top of the rotor arm to the bottom, static balancing probably will be adequate.

9.1 - The required balance weight and its location can be determined either graphically or analytically. The graphical method will be used for static balancing as it only requires drafting aids and does not use trigonometry. An example of the analytical method is given later.

9.2 - The graphical method requires triangles, scale, protractor, pencil, and paper. If polar coordinate paper is used, the protractor can be omitted and the scale replaced by a divider.

10. Static Balancing - *Example*

Single-plane balancing technique. - The unit rotates counterclockwise, the rotating parts weigh 200 000 pounds, and the rotor has six arms equally spaced. Assume a lag angle of 45°. Therefore, the trial weight

Maximum deflections for as-found *condition*

Guide bearing	Shaft deflection, inch	Location, degrees
Upper	0.009	/150
Lower	0.008	/150
Turbine	0.005	/150

should be attached to the rotor arm nearest 225° (180° plus 45°) to the high spot.

$$\text{Location for weight} = 150^\circ + 225^\circ = 375^\circ \text{ or } 15^\circ$$

Rotor arm No. 1 (reference line) at 0° is the nearest arm to 15°.

$$\text{Trial weight} = \frac{\text{weight of rotating parts}}{10\ 000}$$

$$= \frac{200\ 000}{10\ 000} = 20 \text{ pounds}$$

Maximum deflection with the trial weight on top *of rotor arm No. 1*

Guide bearing	Shaft deflection, inch	Location, degrees
Upper	0.006	/200
Lower	0.006	/200
Turbine	0.004	/200

Maximum deflections with the trial weight on bottom *of rotor arm No. 1*

Guide bearing	Shaft deflection, inch	Location, degrees
Upper	0.007	/200
Lower	0.006	/200
Turbine	0.004	/200

Since the shaft deflections with the weight on top of the rotor arm are similar to the deflections with the weight on the bottom, static balancing may be sufficient.

The largest deflection occurs at the upper guide bearing. Because the weight has slightly more effect on the deflection when it is on top of the rotor arm, the balance weight should be attached there.

The upper guide bearing condition from the as-found deflection and of the trial weight on top of rotor arm No. 1 are plotted in figure 8. The as-found deflection is plotted as vector ***OA*** (0.009 inch /150°) with the arrow pointing in the direction of the deflection. Likewise, the deflection with top trial weight is plotted as vector ***OB*** (0.006 /200°). Construct a vector from ***A*** to ***B***. Vector ***AB*** represents the effect of trial weight. The magnitude of ***AB*** and the angle between ***OA*** and ***AB*** are scaled as 0.0069 inch and 42°, respectively.

By studying the triangle of vectors ***OA***, ***OB***, and ***AB*** and applying the vector techniques introduced earlier, a better understanding of this method can be made. Note that the vectors are drawn such that:

$$\boldsymbol{OA} + \boldsymbol{AB} = \boldsymbol{OB}$$

where:

vector ***OA*** is the as-found deflection
vector ***AB*** is the effect of trial weight
vector ***OB*** is the deflection with top trial weight (resultant)

To obtain a static balanced condition requires that the resultant deflection ***OB*** be equal to zero so that:

$$\boldsymbol{OA} + \boldsymbol{AB} = 0$$

Since it is not zero, some unbalance still remains. The following procedure can be used to counter the remaining unbalance.

Beginning at the origin ***O***, draw a vector equal and opposite to ***OA*** and label its head ***C*** (fig. 8). Vector ***OC*** represents the effect required to balance (reduce the unbalance to zero). For simplification, vector AB is shifted to the origin ***O*** and shown as ***OB'***. An effect equal to ***OC*** is needed to balance the unit. To make ***OB'*** equal to ***OC***, ***OB'*** must be rotated 42° to the same angular position as ***OC*** and increased by the ratio ***OC/OB'***.

The magnitude and direction of the effect are directly proportional to the amount and location of the balance weight. Therefore, the balance weight also must be rotated 42° in the direction of rotation and its weight increased by the same ratio ***OC/OB'***.

$$\text{Required ratio} = \frac{OC}{OB'} = \frac{0.009}{0.0069} = 1.3$$

Required length for ***OB'*** = ***OC***

$$\begin{aligned} OC &= (1.3)\, OB' \\ &= 1.3\,(0.0069) \\ &= 0.009 \text{ inch} \end{aligned}$$

$$\begin{aligned} \text{Required balance weight} &= 1.3\,(20) \\ &= 26 \text{ pounds} \end{aligned}$$

Since the required location (42°) does not coincide with a rotor arm, the balance weight must be divided into component weights to place them on the

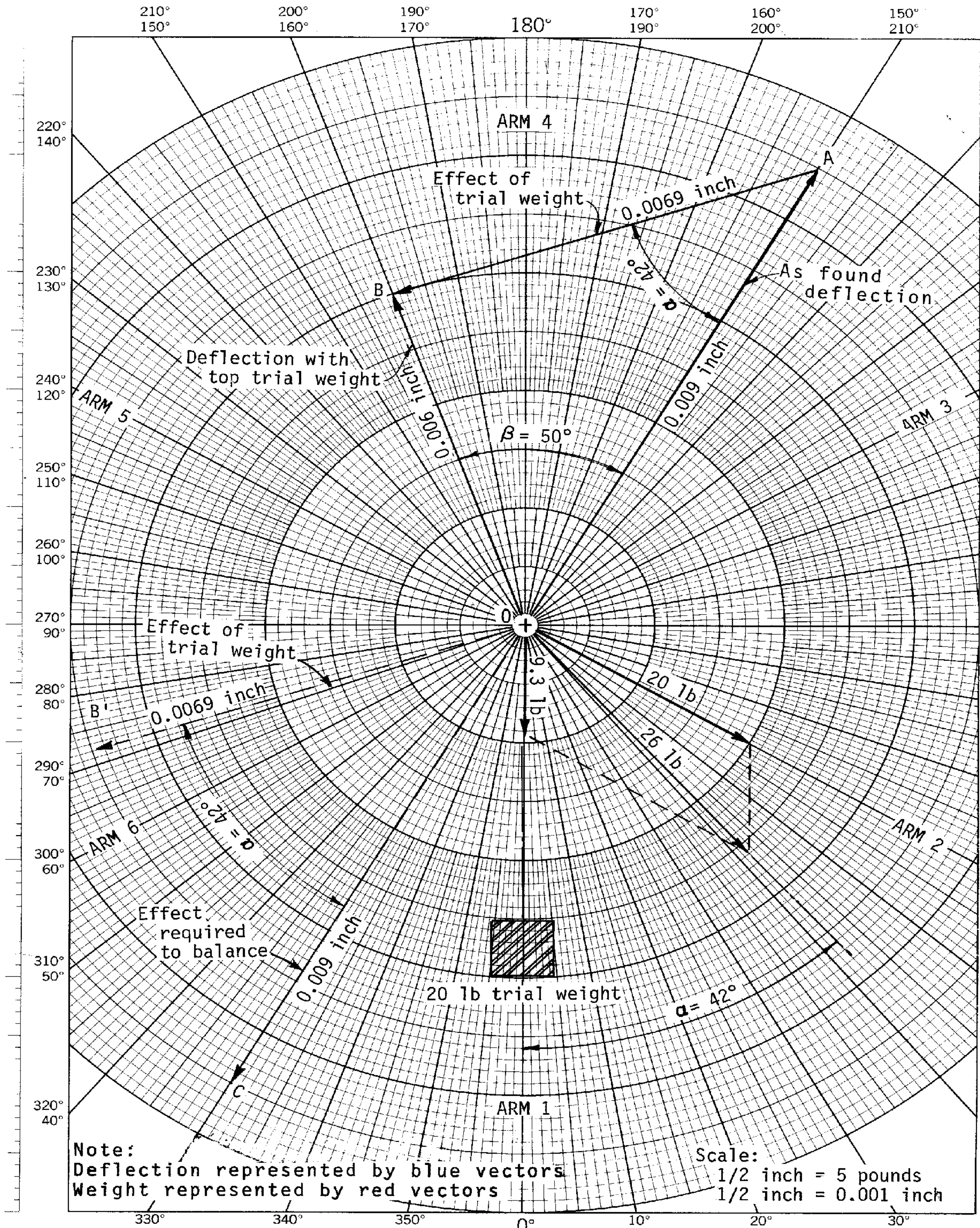


Figure 8.—Static balancing — vectors.

two adjacent arms. Draw the required balance weight as the vector, 26 /42° (fig. 8), and divide that vector into components corresponding to the positions of rotor arms No. 1 and 2. By scaling or reading divisions on the graph paper, the components are:

Weight on rotor arm No. 1 = 9.3 pounds
Weight on rotor arm No. 2 = 20 pounds

Remove the 20-pound trial weight from rotor arm No. 1 and place a 9.3-pound weight on rotor arm No. 1 and a 20-pound weight on rotor arm No. 2. The unit balance should be checked by running at normal speed before the weights are attached permanently to the rotor arms.

11. Dynamic Balancing

In some cases, weight added at one end of the rotor will compound vibration. Here, the *unbalance mass* and the *added weight* form a couple that deflects the rotor.

Figure 9 illustrates a rotor having dynamic unbalance. Weight added at the top of the rotor to counter m_1 will create a static unbalance on the m_2 side of the rotor. To correct dynamic unbalance, weight must be added to both the top and the bottom of the rotor to produce a couple that will counter the effect of the couple causing the vibration. The difficult part of dynamic balancing is determining the correct combinations of weights and locations.

11.1 - The required weights and locations can be determined graphically, analytically, or by a combination of the two. Programs which determine the required combination are available for both computers and programmable calculators. However, the graphical method presented here should be studied to give some insight into the forces involved in dynamic balancing. It may be quite valuable if unusual problems arise which require innovative actions. An example of an analytical dynamic balancing is at the end of the bulletin.

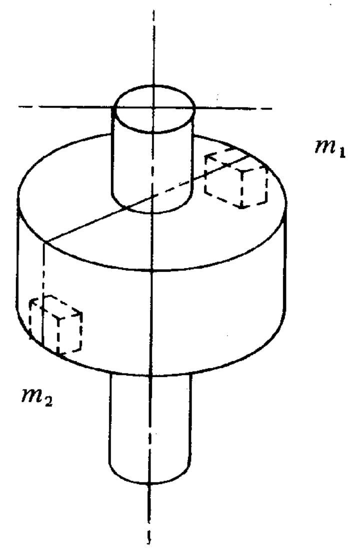


Figure 9.—Dynamic unbalance rotor.

11.2 - The same equipment used for static balancing is used for the graphical method of dynamic balancing. It is noted that the graphical method is a trial-and-error method which intuitively may require judgment and experience.

12. Dynamic Balancing - *Example*

The unit to be balanced rotates counterclockwise, the rotating parts weigh 250 000 pounds, and the rotor has six arms equally spaced.

Maximum deflection for as-found *condition*

Guide bearing	Shaft deflection, inch	Location, degrees
Upper	0.008	/170
Lower	0.007	/0
Turbine	0.006	/0

Plot the maximum deflections for the upper and lower guide bearings on separate sheets and label the heads of the vectors ***A***. Refer to figure 10 and figure 11. Subscripts *t* and *b* refer to upper and lower bearings, respectively.

Add a trial weight at the top of the rotor. In the absence of other information, assume a lag angle of 45°. To counter the unbalance at the upper guide bearing, the weight should be attached to the rotor arm nearest 225° (180° plus 45°) from the high spot.

Location for weight = 170° + 225°
= 395° or 35°

Rotor arm No.2 at 60° is the arm nearest 35°

$$\text{Trial weight} = \frac{\text{weight of rotating parts}}{10\ 000}$$

$$= \frac{250\ 000}{10\ 000} = 25 \text{ pounds}$$

Plot the maximum deflections for the upper and lower guide bearings on the respective sheets and label the heads of the vectors ***B***.

Maximum deflections with the trial weight on top *of rotor arm No. 2*

Guide bearing	Shaft deflection, inch	Location, degrees
Upper	0.003	/240
Lower	0.008	/340
Turbine	0.007	/340

Remove the trial weight from the top of rotor arm No. 2 and put it on the bottom of the rotor arm. The trial weight now should be located to counter the as-found deflection at the lower guide bearing.

Location for weight = 0° + 225° = 225°

Rotor arm No. 5 at 240° is nearest 225°

Maximum deflections with the trial weight on bottom *of rotor arm No. 5*

Guide bearing	Shaft deflections, inch	Location, degrees
Upper	0.009	/180
Lower	0.004	/40
Turbine	0.005	/180

Plot the maximum deflections on the respective sheets and label the heads of the vectors ***C***.

Construct vectors from ***A*** to ***B*** and from ***A*** to ***C*** on both the upper and lower guide bearing sheets. The vectors ***OA, OB***, and ***OC*** are resultant vectors and the vectors ***AB*** and ***AC*** are effect vectors. On figure 10, A_tB_t is the effect of top trial weight (on the rotor top) and A_tC_t is the cross effect of bottom trial weight (on the rotor bottom). On figure 11, A_bB_b is the cross effect of top trial weight (on the rotor top) and A_bC_b is effect of bottom trial weight (on the rotor bottom).

The upper guide bearing has the highest vibration amplitude (shaft deflection). Using the single-plane balancing technique, find the top balance weight required to make the upper guide bearing deflection zero. Start with OC_t rather than OA_t. This way the cross effect of the bottom trial weight is accounted for. From figure 10 and translating A_tB, as OB'_t:

OC_t = 0.009 inch /180°

$A_tB_t = OB'_t$ = 0.0075 /328°

To reduce the deflection to zero, A_tB_t must be equal and opposite to OC_t.

Required angular rotation = 360° - 328°
= 32° counterclockwise

$$\text{Required weight} = \text{trial weight}\ \frac{OC_t}{A_tB_t}$$

$$= 25\left(\frac{0.009}{0.0075}\right) = 30 \text{ pounds}$$

This change will change the cross effect proportionately at the lower guide bearing. The cross effect will be increased by the same ratio OC_t/A_tB_t and rotated 32° ccw (counter-clockwise). From figure 11, the new cross effect is shown as A_bB_x.

$$A_bB_x = A_bB_b \left(\frac{0.009}{0.0075}\right)$$

$$= (0.0018 \text{ inch})\left(\frac{0.0096}{0.0047}\right) = 0.0037 \text{ inch}$$

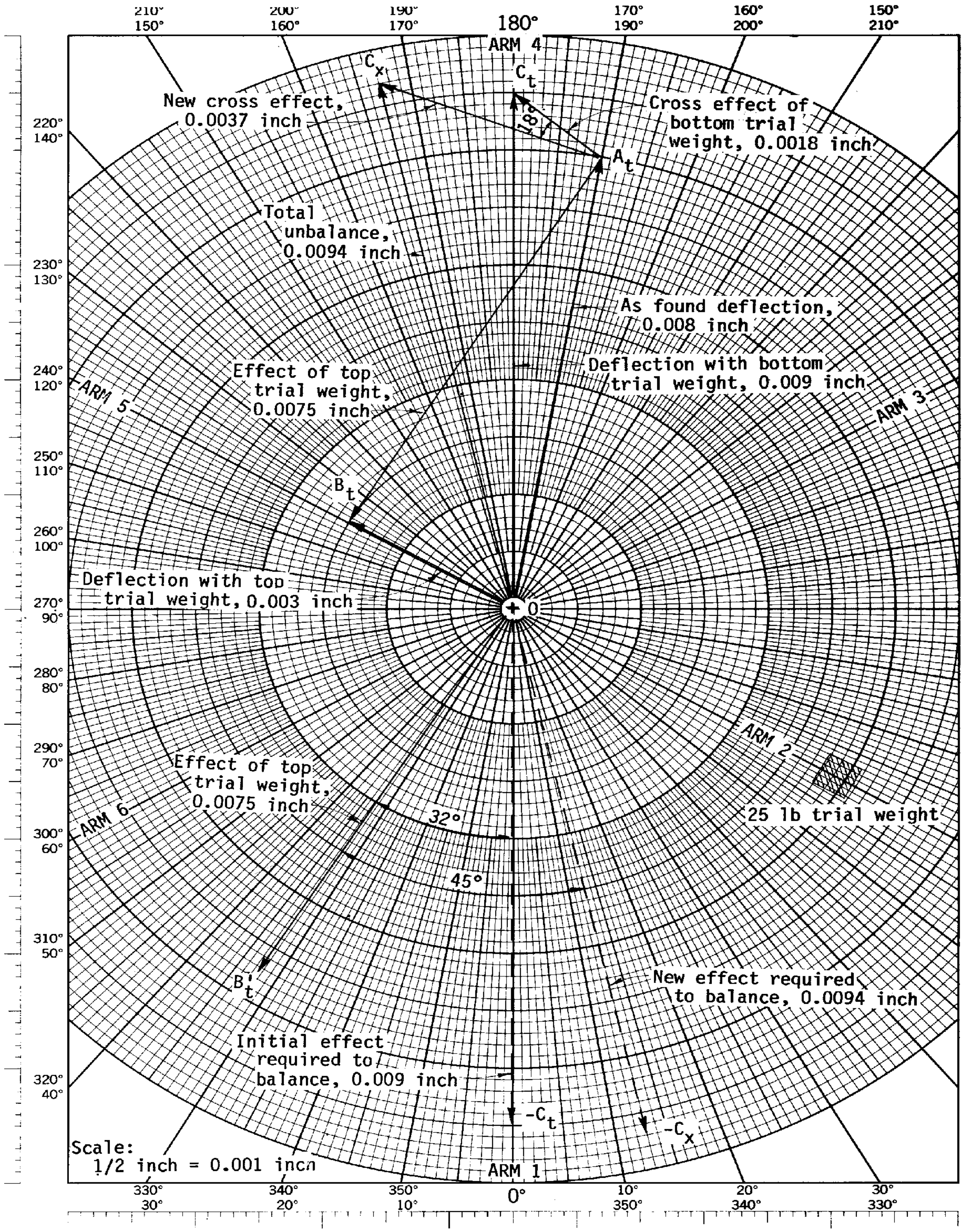


Figure 10. Dynamic balancing – upper guide bearing.

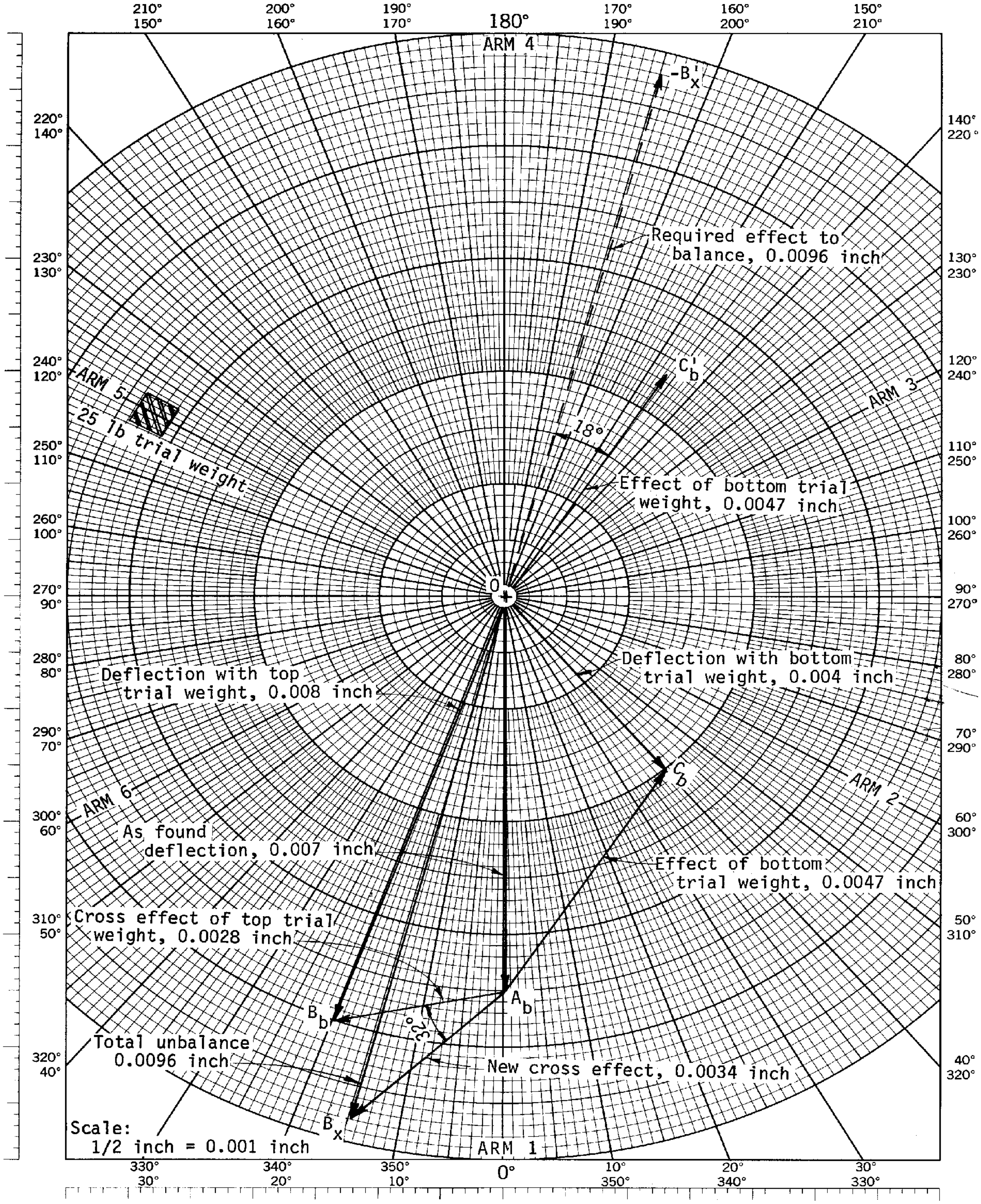


Figure 11.—Dynamic balancing – lower guide bearing.

$\boldsymbol{A_bC_x}$ is rotated 32º ccw from $\boldsymbol{A_bC_b}$.

$\boldsymbol{A_bB_x}$ = 0.0034 inch /<u>281º + 32º</u>
= 0.0034 inch <u>/313º</u> as it would appear with its tail at $\boldsymbol{O}$.

The total unbalance at the lower guide bearing is the vector sum of the original unbalance and the new cross effect.

$$\boldsymbol{OB_x} = \boldsymbol{OA_b} + \boldsymbol{A_bB_x}$$

Use the single-plane technique to find the balance weight that, when added to the bottom of the rotor, will reduce the total unbalance at the lower guide bearing to zero (fig. 11).

$\boldsymbol{OB_x}$ = 0.0096 inch <u>/345º</u>
$\boldsymbol{A_b\,C_b} = \boldsymbol{OC'_b}$ = 0.0047 inch <u>/147º</u>

To reduce the unbalance to zero, $\boldsymbol{A_b\,C_b}$ must be equal and opposite to $\boldsymbol{OB_x}$. Plot $\boldsymbol{-OB_x}$ as $\boldsymbol{OB'_x}$ = 0.0096 <u>/165º</u>. Shift $\boldsymbol{A_b\,C_b}$ to the position shown as $\boldsymbol{OC'_b}$.

Required angular rotation = 165º - 147º
= 18º ccw

$$\text{Required weight} = \text{trial weight}\ \frac{OC'_x}{OB'_t}$$

$$= 25\left(\frac{0.0096}{0.0047}\right) = 51 \text{ pounds}$$

Changing the weight on the bottom of the rotor changes the cross effect proportionately at the upper guide bearing. The cross effect will be increased by the ratio $\boldsymbol{OB'_x/OC'_b}$ and rotated 18º counterclockwise. On figure 10, the new cross effect is shown as $\boldsymbol{A_tC}$.

$$\boldsymbol{A_tC_x} = \boldsymbol{A_tC_t}\left(\frac{0.0096}{0.0047}\right)$$

$$= (0.0018 \text{ inch})\left(\frac{0.0096}{0.0047}\right) = 0.0037 \text{ inch}$$

A_tC_x is rotated 18º ccw from $\boldsymbol{A_tC_t}$

$\boldsymbol{A_tC_x}$ = 0.0037 inch <u>/231º + 18º</u>
= 0.0037 inch <u>/249º</u> as it would appear with its tail at $\boldsymbol{O}$.

The total unbalance at the upper guide bearing is the sum between the vectors $\boldsymbol{OA_t}$ and $\boldsymbol{A_tC_x}$.

$$\text{Total unbalance} = \boldsymbol{A_tC_x} + \boldsymbol{OA_t} = \boldsymbol{OC_x}$$

To balance, again use the single plane technique to find the weight needed to bring the deflection at the upper guide bearing back to zero (fig. 10).

$\boldsymbol{OC_x}$ = 0.0094 inch <u>/193º</u>
$\boldsymbol{A_tB_t} = \boldsymbol{OB'_t}$ = 0.0075 inch <u>/328º</u>

To bring the unbalance back to zero, $\boldsymbol{A_tB_t}$ must be equal and opposite $\boldsymbol{OC_x}$. Plot $\boldsymbol{OC_x}$ as

$\boldsymbol{OC_x'}$ = 0.0094 <u>/13º</u>. Recall that $\boldsymbol{A_tB_t} = \boldsymbol{OB'_t}$.

Required angular rotation = 13º - 328º +360º
= 45º ccw

$$\text{Required weight} = \text{trial weight}\ \frac{OC'_x}{OB'_t}$$

$$= 25\left(\frac{0.0094}{0,0075}\right) = 31 \text{ pounds}$$

Now a balance check can be made on the lower guide bearing. Again, the addition of this new weight will change the cross effect proportionately at the lower guide bearing. Referring to figure 12:

$$\boldsymbol{A_bB_{xx}} = \boldsymbol{A_bB_b}\left(\frac{0.0096}{0.0047}\right) \;/\; 281^\circ + 45^\circ$$

$$= (0.0028 \text{ inch})\left(\frac{0.0096}{0.0047}\right) \;/\; 326^\circ$$

$$= 0.0035 \text{ inch} \;/\; 326^\circ$$

The total unbalance at the lower guide bearing is the difference between vectors $\boldsymbol{OB_{xx}}$ and $\boldsymbol{OB_x}$.

Total unbalance = $\boldsymbol{OZ} = \boldsymbol{OB_{xx}} - \boldsymbol{OB_x}$
= 0.0009 inch <u>/40º</u>

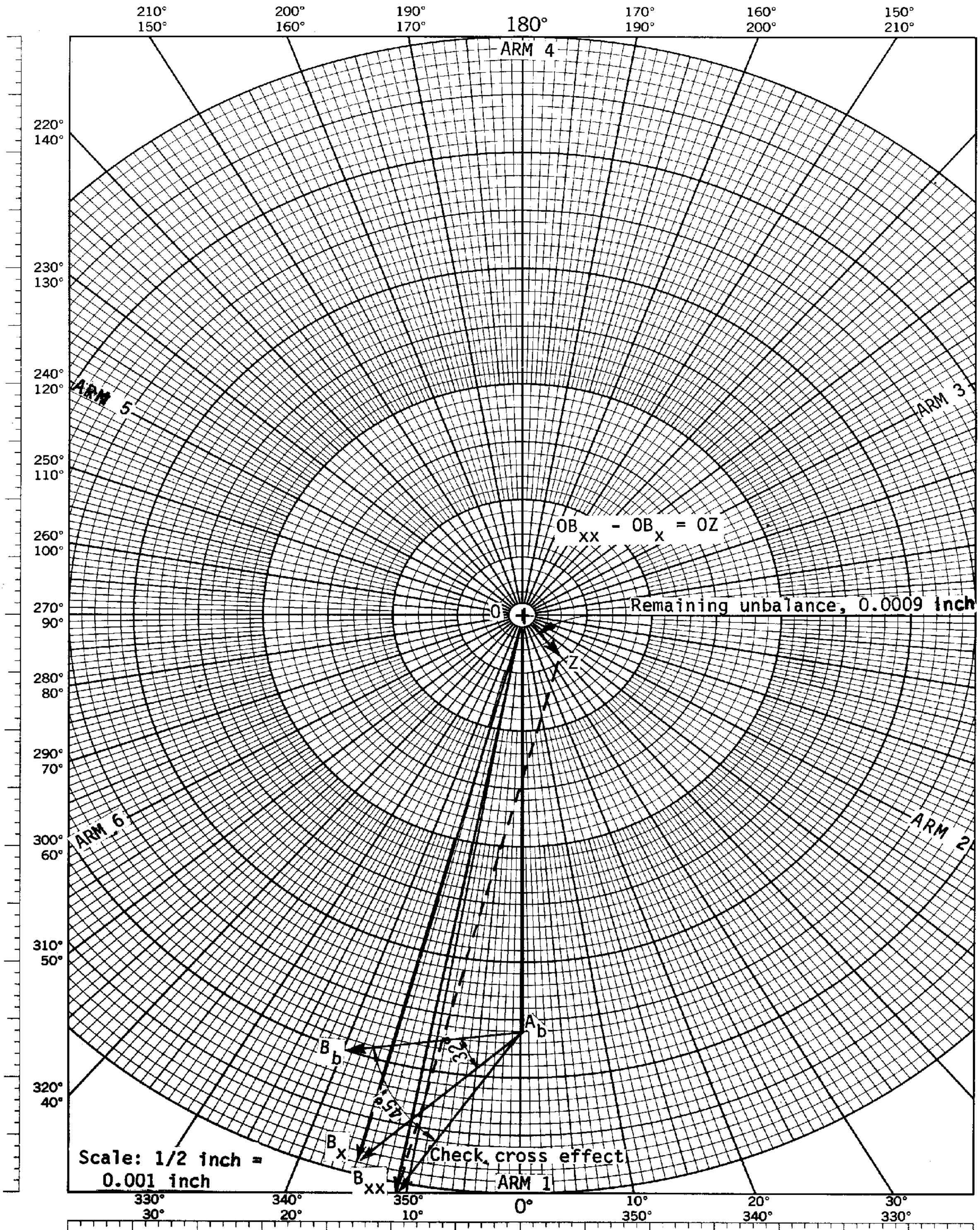


Figure 12.–Dynamic balancing – lower guide bearing, check.

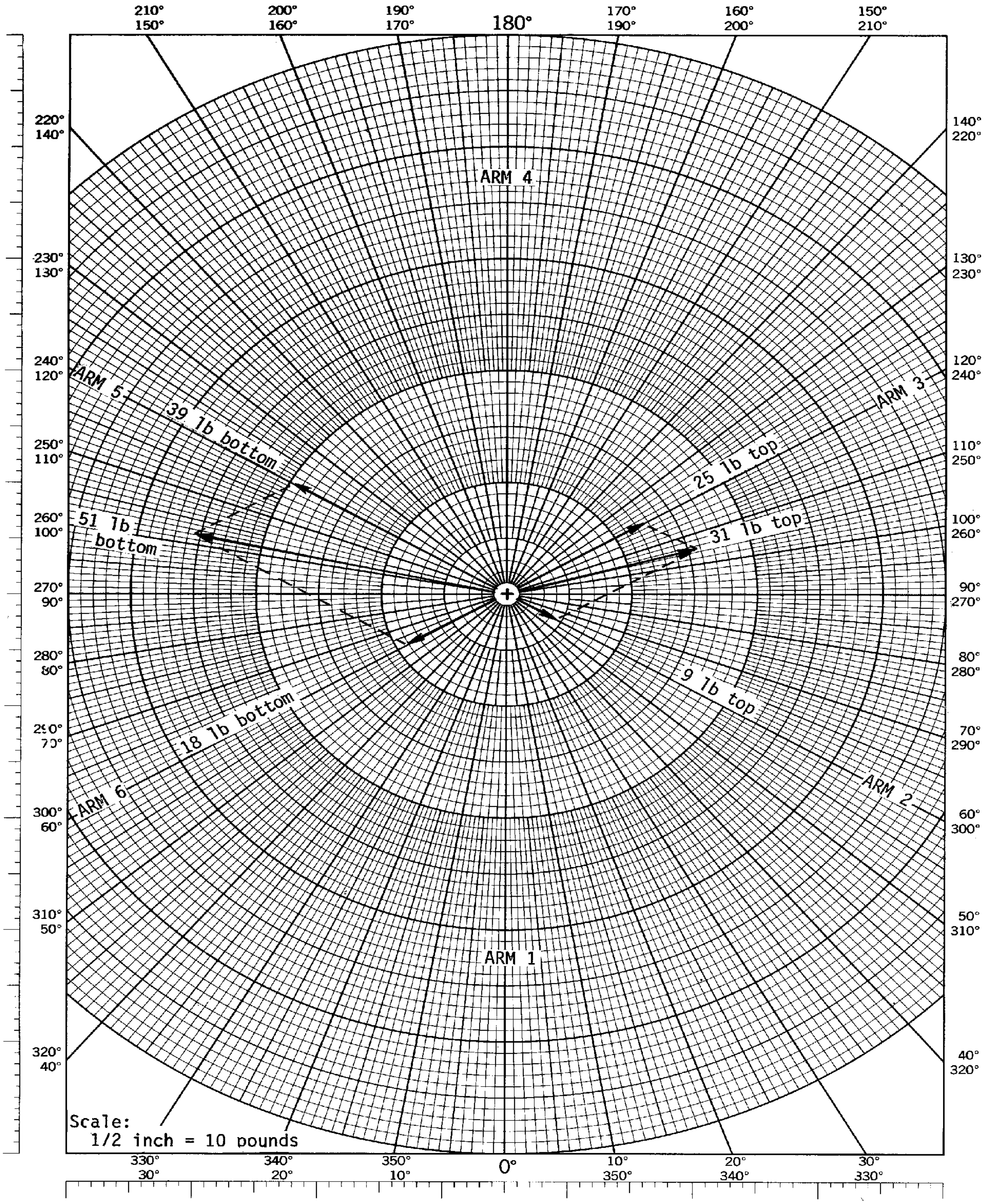


Figure 13.–Dynamic balancing – balance weight distribution.

Draw the desired balance weights as vectors on polar coordinate paper and resolve them into components corresponding to the adjacent rotor arms as shown on figure 13.

Place the weights on the appropriate rotor arms and obtain another set of deflection readings. Some adjustment may be required due to data or analysis inaccuracies.

13. Analytical Static Balancing - *Example*

Paragraph 10, Static Balancing - *Example*, will be used again for analytical static balancing.

Data

Weight of rotating parts . . . . 200 000 pounds
Number of rotor arms . . . . 6 equally spaced
Unit rotation . . . . counterclockwise
Trial weight . . . . 20 pounds

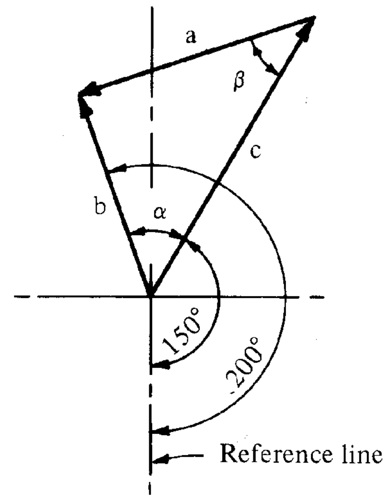
Shaft deflections . . . . *as found*
Upper guide bearing . . . . 0.009 inch /150°
Lower guide bearing . . . . 0.008 inch /150°
Turbine guide bearing . . . . 0.005 inch /150°

Shaft deflections - trial weight on *top* of rotor arm No. 1
Upper guide bearing . . . . 0.006 inch /200°
Lower guide bearing . . . . 0.006 inch /200°
Turbine guide bearing . . . . 0.004 inch /200°

Shaft deflections - trial weight on *bottom* of rotor arm No. 1
Upper guide bearing . . . . 0.007 inch /200°
Lower guide bearing . . . . 0.006 inch /200°
Turbine guide bearing . . . . 0.004 inch /200°

The trial weight has more effect on the shaft deflections when it is on top of the rotor arm; therefore, the balance weight should be attached on top.

Figure 14 shows the *as-found* unbalance at the upper guide bearing as well as the effect produced by positioning the trial weight on top of rotor arm No. 1.



a = Effect of trial weight

b = Unbalance with trial weight, 0.006 inch /200

c = As found unbalance 0.009 inch /150

Figure 14.–Conditions at the upper guide bearing.

To balance the unit, the balance weight must have an effect equal to the *as-found* unbalance and in the opposite direction.

Required balance weight = weight causing unbalance

It was assumed that the machine is linear; i.e., the vibration amplitudes are in proportion to the forces causing them:

$$\frac{\boldsymbol{C}}{\boldsymbol{A}} = \frac{\text{as - found unbalance}}{\text{effect of trial weight}}$$

$$= \frac{\text{weight causing unbalance}}{\text{trial weight}}$$

The effect of trial weight ***A*** can be determined using the *law of cosines.*

$$a^2 = b^2 + c^2 - 2\,bc \cos \alpha$$

$$a = [b^2 + c^2 - 2\,bc \cos \ \alpha]^{0.5}$$

$$a = [0.006^2 + 0.009^2 - 2\,(0.006)(0.009)$$
$$\mathrm{Cos}\,(200^{\circ} - 150^{\circ})]^{0.5}$$

$$a = [0.000117 - 2(0.000054)\ 0.64279]^{0.5}$$

$$a = [47.6 \text{ x } 10\text{-}6]^{0.5} = 0.0069 \text{ inch}$$

The fact that the required balance weight equals the weight causing unbalance:

Required balance weight = (trial weight) (c/a)
= 20 (0.009/0.0069)
= 26 pounds

The angle by which the effect must be changed, equals the angle the weight must be rotated. Solve for using the *law of sines*:

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta}$$

$$\sin \beta = (b/a) \sin \alpha$$
$$= (0.006/0.0069) \sin (200^{\circ} - 150V)$$
$$\sin \beta = (0.006/0.0069))0.76604) = 0.66612$$

$$\beta = 41.8^{\circ}$$

The required location for the weight is 41.8º counterclockwise from rotor arm No. 1, the location of the trial weight. The required location does not coincide with a rotor arm so the weight must be divided for placement on adjacent rotor arms.

The required weights are obtained using the sine law (fig. 15):

$$\frac{W}{\sin \ \gamma} = \frac{W_1}{\sin \alpha} = \frac{W_2}{\sin \beta}$$

where:

W = 26 pounds
β = 41.8º

The angle between rotor arms is 60º:
thus:

$$\alpha + \beta = 60^{\circ}$$
$$\alpha = 60^{\circ} - \beta = 60^{\circ} - 41.8^{\circ}$$
$$= 18.2^{\circ}$$

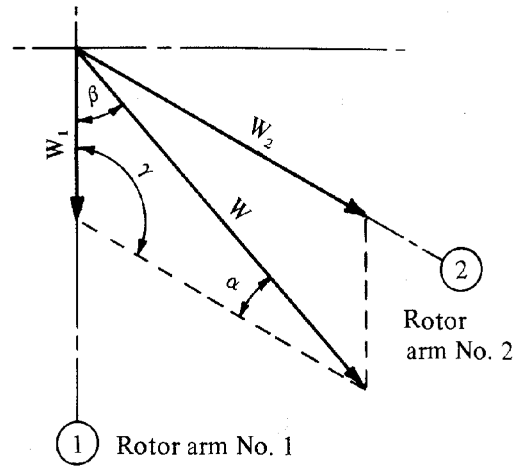


Figure 15.–Weight distribution on rotor arms.

The sum of triangle angles must equal 180º

$$\alpha + \beta + \gamma = 180^{\circ}$$
$$\gamma = 180^{\circ} - \alpha - \beta$$
$$= 180^{\circ} - 18.2^{\circ} - 41.8^{\circ} = 120^{\circ}$$

The weights for rotor arms No 1 and 2 now can be determined.

$$W_1 = \frac{W \sin \alpha}{\sin \gamma} = \frac{26 \sin 18.2^{\circ}}{\sin 120^{\circ}}$$
$$= 9.3 \text{ pounds}$$

$$W_1 = \frac{W \sin \beta}{\sin \gamma} = \frac{26 \sin 41.8^{\circ}}{\sin 120^{\circ}}$$

$$= 20.0 \text{ pounds}$$

Weights W_1 and W_2 are attached to the respective rotor arms.

14. Analytical Dynamic Balancing - *Example*

Paragraph 12, Dynamic Balancing - *Example*, will be used again for analytical dynamic balancing.

Data

Weight of rotating parts . . . . 250 000 pounds
Number of rotor arms . . . . .6 equally spaced
Unit rotation . . . . counterclockwise
Trial weight . . . . 25 pounds

Shaft deflections . . . . as found
Upper guide bearing . . . . 0.008 inch /170º
Lower guide bearing . . . . 0.007 inch /0º
Turbine guide bearing . . .. 0.006 inch /0º

Shaft deflections - trial weight on top of rotor arm No. 2
Upper guide bearing . . . . 0.003 inch /240º
Lower guide bearing . . . . 0.008 inch /340º
Turbine guide bearing . . . . 0.007 inch /140º

Shaft deflections - trial weight on bottom of rotor arm No. 5
Upper guide bearing . . . . 0.009 inch /180º
Lower guide bearing . . . . 0.004 inch /40º
Turbine guide bearing . . . . 0.005 inch /180º

Two more vector operations are needed for analytical dynamic balancing; i.e., vector division and multiplication.

To divide one vector by another vector, divide the magnitudes and subtract the angles.

Example: To *divide* a vector of 0.006 inch at 30º by a vector of 0.005 at 150º, divide magnitudes

0.006 inch/0.005 inch = 1.200

and subtract angles

30º - 150º = - 120º

To change the negative angle to a positive angle, add 360º. The result is 1.200 at - 120º or 1.200 at +240º.

To *multiply* one vector by another, multiply the magnitudes and add the angles.

Example: To multiply a vector of 0.004 inch at 180º by a vector of 0.010 inch at 45º, multiply magnitudes

0.004(0.010) = 0.00004

and add angles

180º + 45º = 225º

(0.004 inch /180º)(0.010 inch /45º)
= 0.00004 /225º

The unit was assumed to be linear so the required weights $\boldsymbol{W_{rt}}$ and $\boldsymbol{W_{rb}}$ and locations will be proportional to the trial weights, that is:

$$\boldsymbol{W_{rt}} = \boldsymbol{W_t}$$

$$\boldsymbol{W_{rb}} = \boldsymbol{W_b}$$

where:

θ and ϕ are vector operators (having both magnitude and direction, and
t and *b* refer to top and bottom subscripts, respectively.

To balance the unit, the values of and must be determined.

Using the notation in the graphical method where ***A*** refers to the *as-found* deflection, ***B*** refers to deflection with top trial weight, and ***C*** refers to deflection with bottom trial weight, the vectors are:

$\boldsymbol{OA_t}$ = 8 /170º $\boldsymbol{OC_t}$ = 9 /180º $\boldsymbol{OB_t}$ = 3 /240º
$\boldsymbol{OA_b}$ = 7 /0º $\boldsymbol{OC_b}$ = 4 /40º $\mathbf{OB_b}$ = 8/340º

For convenience, magnitudes are in mils rather than in decimals (thousandths of an inch).

As shown on figures 16 and 17:

$OB_t = \boldsymbol{OA_t} + \boldsymbol{A_t}\,\mathbf{B_t}$ $\boldsymbol{OB_b} = \boldsymbol{OA_b} + A_b\,B_b$
$\boldsymbol{OC_t} = \boldsymbol{OA_t} + \boldsymbol{A_t\,C_t}$ $\boldsymbol{OC_b} = \boldsymbol{OA_b} + \boldsymbol{A_b\,C_b}$

To balance the unit, the combined effect of the balance weights must be equal and opposite to the *as-found* unbalance at both the upper and lower guide bearings. Since vibration amplitudes are proportional to the forces causing them, $\boldsymbol{A_t\,B_t}$ and $\boldsymbol{A_bB_b}$ are proportional to $\boldsymbol{W_t}$ and $\boldsymbol{A_bC_b}$ and $\boldsymbol{A_t\,C_t}$ are proportional to $\boldsymbol{W_b}$. Also, the required balance weight is proportional to the as-found unbalance; i.e., $\boldsymbol{W_{rt}}$ is proportional to $\boldsymbol{OA_t}$ and W_{rb} is proportional to $\boldsymbol{OA_b}$.

$$-\boldsymbol{OA_t} = \theta\,\boldsymbol{A_tB_t} + \phi\,\boldsymbol{A_tC_t}$$

$$-\boldsymbol{OA_b} = \theta\,\boldsymbol{A_bB_b} + \phi\,\boldsymbol{A_bC_b}$$

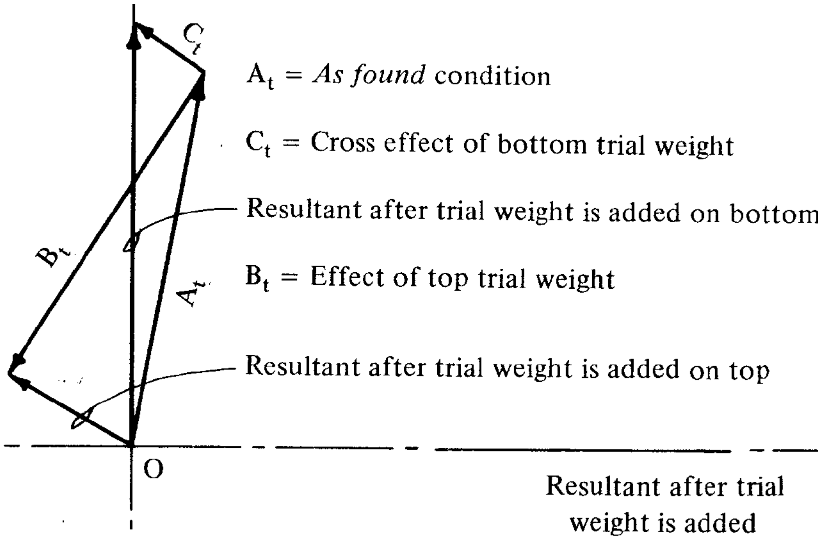


Figure 16.—Deflections at upper guide bearing.

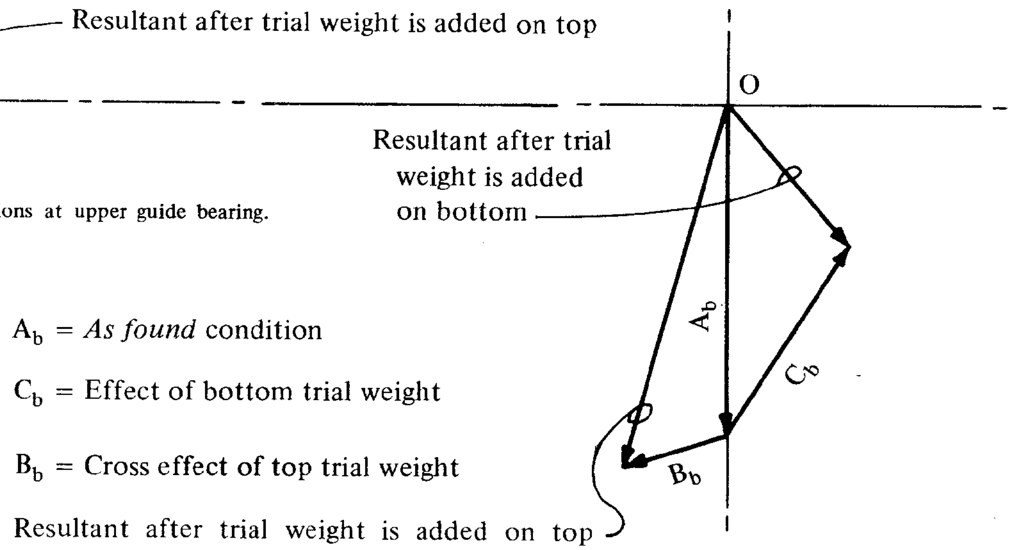


Figure 17.—Deflections at lower guide bearing.

From these equations, the vector operators are determined by

$$\theta = \frac{(\boldsymbol{OA}_t \cdot \boldsymbol{A}_b\boldsymbol{C}_b) - (\boldsymbol{OA}_b \cdot \boldsymbol{A}_t\,\boldsymbol{C}_t)}{(\boldsymbol{A}_b\boldsymbol{B}_b \cdot \boldsymbol{A}_t\,\boldsymbol{C}_t) - (\boldsymbol{A}_t\,\boldsymbol{B}_t \cdot \boldsymbol{A}_b\boldsymbol{C}_b)}$$

$$\phi = \frac{(\boldsymbol{OA}_b \cdot \boldsymbol{A}_t\boldsymbol{C}_t) - (\boldsymbol{OA}_t \cdot \boldsymbol{A}_b\,\boldsymbol{C}_b)}{(\boldsymbol{A}_b\boldsymbol{B}_b \cdot \boldsymbol{A}_t\,\boldsymbol{C}_t) - (\boldsymbol{A}_t\,\boldsymbol{B}_t \cdot \boldsymbol{A}_b\boldsymbol{C}_b)}$$

Remember, these are vector equations.

$\boldsymbol{OA}_t = 8\ \underline{/170^\circ}$

$\boldsymbol{OA}_b = 7\ \underline{/0^\circ}$

$\boldsymbol{OB}_t = \mathbf{OA}_t + \boldsymbol{A}_t\,\boldsymbol{B}_t = 3\ \underline{/240^\circ}$

$\boldsymbol{OB}_b = \boldsymbol{OA}_b + \boldsymbol{A}_b\,\boldsymbol{B}_b = 8\ /340^\circ$

$\boldsymbol{OC}_t = \boldsymbol{OA}_t + \boldsymbol{A}_b\,\boldsymbol{C}_b = 9\ /180^\circ$

$\boldsymbol{OC}_b = \boldsymbol{OA}_b + \boldsymbol{A}_t\,\boldsymbol{C}_t = 4\ /40^\circ$

The effects of the trial weights are computed using vector subtraction or can be scaled:

$\boldsymbol{A}_t\,\boldsymbol{B}_t = \boldsymbol{OB}_t - \boldsymbol{OA}_t = 3\ \underline{/240^\circ} - 8\ \underline{/170^\circ}$

Magnitude of $\boldsymbol{A}_t\,\boldsymbol{B}_t$

$$= [\ (3 \sin 240 - 8 \sin 1\ 70)^2 + (3 \cos 240 - 8 \cos 1\ 70)^2\ ]^{0.5}$$
$$= [(-\ 3.99)^2 + (6.38)^2\ ]^{0.5} = 7.52$$

Angle of:

$$\boldsymbol{A_tB_t} = \text{arc tan}\,\frac{-3.99}{6.38} = \text{arc tan} - 0.6254$$
$$= 328^\circ$$
$$\boldsymbol{A_tB_t} = 7.52\ \underline{/328^\circ}$$

$$\boldsymbol{A_bB_b} = \boldsymbol{OB_b} - \boldsymbol{OA_b} = 8\ \underline{/340^\circ} - 7\ \underline{/0^\circ}$$
$$= 2.78\ \underline{/280.7^\circ}$$
$$\boldsymbol{A_tC_t} = \boldsymbol{OC_t} - \boldsymbol{OA_t} = 9\ \underline{/180^\circ} - 8\ \underline{/170^\circ}$$
$$= 1.79\ \underline{/231.1^\circ}$$
$$\boldsymbol{A_bC_b} = \boldsymbol{OC_b} - \boldsymbol{OA_b} = 4\ \underline{/40^\circ} - 7\ \underline{/0^\circ}$$
$$= 4.70\ \underline{/146.8^\circ}$$

Now, use vector multiplication:

$$\boldsymbol{OA_t} \bullet \boldsymbol{A_bC_b} = 8\ \underline{/170^\circ} \bullet 4.70\ \underline{/146.8^\circ}$$
$$= 8 \bullet 4.70\ \underline{/170^\circ + 146.8^\circ}$$
$$= 37.60\ \underline{/316.8^\circ}$$
$$\boldsymbol{OA_b} \bullet \boldsymbol{A_tC_t} = 7\ \underline{/0^\circ} \bullet 1.79\ \underline{/231.1^\circ}$$
$$= 12.53\ \underline{/231.^\circ}$$
$$\boldsymbol{OA_b} \bullet \boldsymbol{A_tB_t} = 7\ \underline{/0^\circ} \bullet 7.52\ \underline{/328^\circ}$$
$$= 52.64\ \underline{/328^\circ}$$
$$\boldsymbol{OA_t} \bullet \boldsymbol{A_bB_b} = 8\ \underline{/170^\circ} \bullet 2.78\ \underline{/280.7^\circ}$$
$$= 22.24\ \underline{/90.7^\circ}$$
$$\boldsymbol{A_bB_b} \bullet \boldsymbol{A_tC_t} = 2.78\ \underline{/280.7^\circ} \bullet 1.79\ \underline{/231.1^\circ}$$
$$= 4.98\ \underline{/151.8^\circ}$$
$$\boldsymbol{A_tB_t} \bullet \boldsymbol{A_bC_b} = 7.52\ \underline{/328^\circ} \bullet 4.70\ \underline{/146.8^\circ}$$
$$= 35.34\ \underline{/114.8^\circ}$$

Vector subtraction is now used to determine the numerator and the denominator:

$$(\boldsymbol{OA_t} \bullet \boldsymbol{A_bC_b}) - (\boldsymbol{OA_b} \bullet \boldsymbol{A_tC_t})$$
$$= 37.60\ \underline{/316.8^\circ} - 12.53\ \underline{/231.1^\circ}$$
$$= 38.73\ \underline{/335.6^\circ}$$
$$(\boldsymbol{OA_b} \bullet \boldsymbol{A_tB_t}) - (\boldsymbol{OA_t} \bullet \boldsymbol{A_bB_b})$$
$$= 52.64\ \underline{/328^\circ} - 22.24\ \underline{/90.7^\circ}$$
$$= 67.31\ \underline{/311.9^\circ}$$
$$(\boldsymbol{A_b\,B_b} \bullet \boldsymbol{A_tC_t}) - (\boldsymbol{A_tB_t} \bullet \boldsymbol{A_bC_b})$$
$$= 4.98\ \underline{/151.8^\circ} - 35.34\ \underline{/114.8^\circ}$$
$$= 31.51\ \underline{/289.3^\circ}$$

$$\theta = \frac{38.73\ \underline{/335.6}}{31.54\ \underline{/289.3}} = \frac{38.73}{31.51}\ \underline{/335.6 - 289.3}$$
$$= 1.23\ \underline{/46.3}$$
$$\phi = \frac{67.31\ \underline{/311.9}}{31.51\ \underline{/289.3}} = \frac{67.31}{31.51}\ \underline{/311.9 - 289.3}$$
$$= 2.14\ \underline{/22.6}$$

$$W_{rt} = \theta W_t = 1.23\ \underline{/46.3^\circ} \bullet 25\ \underline{/60^\circ}$$
$$= 30.75\ \underline{/106.3^\circ}$$

$$\mathrm{W_{rb}} = \phi \mathrm{W}_b = 2.14\ \underline{/12.6^\circ} \bullet 25\ \underline{/240^\circ}$$
$$= 53.5\ \underline{/262.6^\circ}$$

Because the required locations do not coincide with rotor arms, the weights must be divided for attachment on the adjacent rotor arms.

The top weight is proportioned between rotor arms No. 2 and 3 using the sine law (see fig. 18).

$$\frac{W_{rt}}{\sin \gamma_t} = \frac{W_2}{\sin \beta_t} = \frac{W_3}{\mathrm{Sin}\ \alpha_t}$$

where:

$W_{rt} = 30.75$ pounds
$\alpha_t = 46.3^\circ$
$\alpha_t + \beta_t = 60^\circ$
$\beta_t = 60^\circ - \alpha_t = 60^\circ - 46.3^\circ = 13.7$
$\alpha_t + \beta_t + \gamma_t = 180^\circ$
$\gamma_t = 180^\circ - (\alpha_t + \beta_t) = 180^\circ - 60^\circ = 120^\circ$

$$W_2 = \mathrm{W}_{rt}\frac{\sin \beta_t}{\sin \gamma_t} = 30.75\frac{\sin 13.7^\circ}{\sin 120^\circ}$$
$$= 8.4 \text{ pounds}$$
$$W_3 = \mathrm{W}_{rt}\frac{\sin \alpha_t}{\sin \gamma_t} = 30.75\frac{\sin 46.3^\circ}{\sin 120^\circ}$$
$$= 25.7 \text{ pounds}$$

Similarly, the bottom weight is proportioned between arms No. 5 and 6 (see fig. 19).

$$\frac{W_{rb}}{\sin \gamma_b} = \frac{W_5}{\sin \beta_b} = \frac{W_6}{\mathrm{Sin}\ \alpha_b}$$

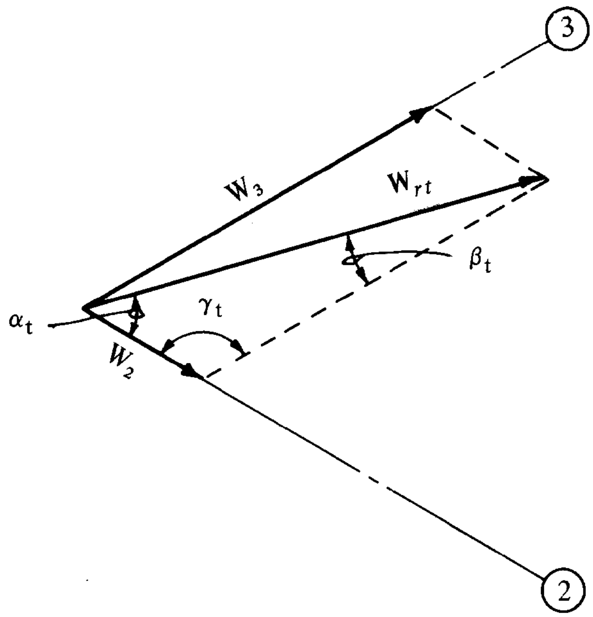


Figure 18.–Divided weight for upper rotor arms No. 2 and 3.

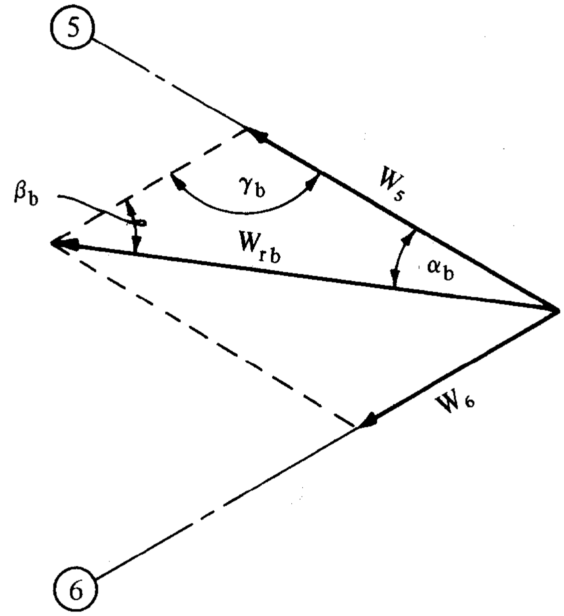


Figure 19.–Divided weight for lower rotor arms No. 5 and 6.

where:

W_{rb} = 53.5 pounds
$\alpha_b = 22.6°$
$\alpha_b + \beta_b = 60°$
$\beta_b = 60° - \alpha_b = 60° - 22.6° = 37.4°$
$\alpha_b + \beta_b + \gamma_b = 180°$
$\gamma_b = 180° - (\alpha_b + \beta_b) = 180° - 60° = 120°$

$$W_5 = W_{rb} \frac{\sin \beta_b}{\sin \gamma_b} = 53.5 \frac{\sin 37.4°}{\sin 120°}$$

= 37.5 pounds

$$W_6 = W_{rb} \frac{\sin \alpha_b}{\sin \gamma_b} = 53.5 \frac{\sin 22.6°}{\sin 120°}$$

= 23.7 pounds

The weights are attached to the respective rotor arms.

The literature available has many cases in the analyses of pump, turbine, and pump-turbine vibrations. As noted, one must first investigate the cause of vibrations and oscillations excited by such conditions as vortex shedding, draft tube surges, penstock pressure fluctuations, electrical system, and mechanics.

MISSION STATEMENTS

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The mission of the Bureau of Reclamation is to manage, develop, and protect water and related resources in an environmentally and economically sound manner in the interest of the American public.

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